Geometry Paper IV March/April 2016.

① The direction cosine of z-axis are.....

a) 1, 1, 1　　b) 1, 0, 0
c) 0, 1, 0　　d) 0, 0, 1

② The number of arbitary constant in the eq^n
$Ax + By + Cz + D = 0$ is ....

a) 4　(b) 3　(c) 2　(d) 1

3 The intercept on z-axis of the plane
$x + y + 2z = 2$ is.

a) 1　(b) 2　(c) 3　(d) 4

④ The value of λ for which the lines
$\frac{x-1}{1} = \frac{y-2}{\lambda} = \frac{z+1}{-1}$ and $\frac{x-1}{-\lambda} = \frac{y+1}{2} = \frac{z-2}{1}$
are perpendicular to each other is:

(a) -1　(b) 1　(c) 0　(d) 2

⑤ The lines L_1 and L_2 intersect the shortest distance between them is .....

(a) positive　　(b) Negative
(c) zero　　d) Infinity.

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⑥ The equation of the sphere on the join of (x_1, y_1, z_1) and (x_2, y_2, z_2) as the dimeter:

a) $(x-x_1)+(y-y_1)(y-y_2)+(z-z_1)(z-z_2)=0$

b) $\sqrt{(x-x_1)(x-x_2)}+\sqrt{(y-y_1)(y-y_2)}+\sqrt{(z-z_1)(z-z_2)}=0$

c) $\frac{x-x_1}{x-x_2}+\frac{y-y_2}{y-y_2}+\frac{z-z_1}{z-z_2}=0$

d) None of the above.

⑦ The equation of the sphere passing through $(0,0,0)$, $(a,0,0)$, $(0,b,0)$, $(0,0,c)$ is

a) $x^2+y^2+z^2+2ax+2by+2cz=0$

b) $x^2+y^2+z^2-2ax-2by-2cz=0$

c) $x^2+y^2+z^2+ax+by+cz=0$

d) $x^2+y^2+z^2-ax-by-cz=0$

⑧ A Surface generated by a straight line which passes through a fixed point and satisfied one more condition; for instance it may intersect a given curve or touch a given surface is called.

a) Cylinder b) Cone

c) Sphere d) Conicoid

⑨ Section of a right circular by any plane perpendicular to it's axis is called ....

a) Normal b) a plane section

c) radius of cylinder d) None of these.

① Prove that the projection of a segment AB on a line CD is $AB \cos\theta$, where θ is the angle between the lines AB and CD.

ns: Let the planes through the point A and B perpendicular to the line CD meet the line CD in A', B' respectively so that $A'B'$ is the projection of AB through the point A draw a line $AP \parallel CD$ to meet the plane through the point B at P.

Now $AP \parallel CD$

$\therefore \angle PAB = \theta$

Also BP lies in the plane which is perpendicular to AP

$\angle APB = 90^\circ$

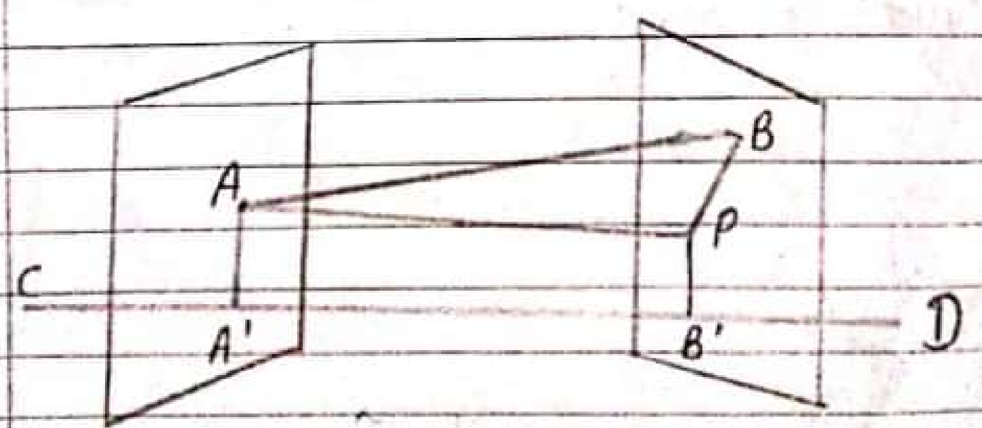
$AP = AB \cos\theta$

clearly $A'B'PA$ is a rectangle implying that

$AP = A'B'$

Hence

$A'B' = AB \cos\theta$.



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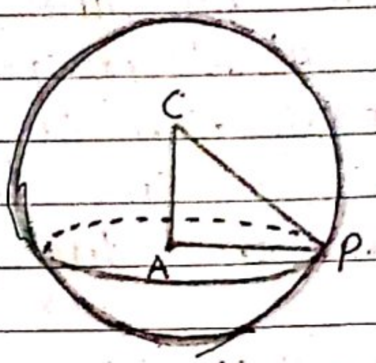
Show that the plane section of the sphere is circle. [octo-Nov-2016]

Ans: Let the sphere and the plane intersects. Suppose that C is the centre of the sphere,



P any point on the plane section and A the foot of the perpendicular drawn from C on the plane section.

$$CP^2 = CA^2 + AP^2$$

or $$AP^2 = CP^2 - CA^2 \quad \text{------} \ (1)$$

Now CP is constant, being the radius of the sphere, for all P. Also the point C and A are fixed for all P. Thus CP and CA are consta[nt]. Then eq^n (1) implies that AP = constant.

Hence the point P moves in a plane in such a way that is its distance from a ~~distan~~ fixed point A is constant. Hence the locus of P is a circle with centre at A and radius equal to AP.

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- find the equation of the plane through the three points $(1,1,1)$, $(1,-1,1)$, $(-7,-3,-5)$

S: The general equation of the plane through $P(1,1,1)$ is

$$a(x-1)+b(y-1)+c(z-1)=0 \quad \text{--- ①}$$

It will pass through Q and R, if
$Q=(1,-1,1)$
in equation ① become.

$$a(1-1)+b(-1-1)+c(1-1)=0$$
$$= -2b \quad \text{--- ii}$$

It will passing through $R(-7,-3,-5)$

$$a(-7-1)+b(-3-1)+c(-5-1)=0$$
$$= -8a \mp 4b - 6c = 0$$
$$= 8a+4b+6c=0 \quad \text{--- iii)}$$

Solving eq^n ii & iii

$$\begin{vmatrix} x & y & z \\ 0 & -2 & 0 \\ 8 & 4 & 6 \end{vmatrix} = (-12-0)x-(0-0)y+(0+16)z$$
$$= -12x + 16z$$
$$= \frac{a}{-12} = \frac{c}{16}$$
$$= \frac{a}{-3} = \frac{c}{4}$$

Puting in eq^n ①
$a=-3$ and $c=4$

$$-3(x-1)+0(y-1)+4(z-1)=0$$
$$= -3x+3+4z-4=0$$
$$= -3x+4z-1$$
$$3x \mp 4z+1$$

$$\boxed{3x \mp 4z+1}$$

As the required equation.

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Q3 Find the equations of the line passing through a given point $A(x_1, y_1, z_1)$ and having direction cosines l, m, n. (oclomber Nove 2016)

Ans: let $p(x, y, z)$ be a point on the given line and let $Ap = r$

Projecting the segment Ap on the co-ordinate axes, we obtain

$$x - x_1 = lr, \quad y - y_1 = mr, \quad z - z_1 = nr$$

So that for all points (x, y, z) on the given line, we have

$$x = x_1 + lr, \quad y = y_1 + mr, \quad z = z_1 + nr.$$

Thus, the set of points on the given line is

$$[(x_1 + lr, y_1 + mr, z_1 + nr)]:$$

r being any number

In case none of l, m, n is zero, we have

$$\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n} = r$$

Thus, if $l \neq 0, m \neq 0, n \neq 0$ or equivalently $lmn \neq 0$.

$$\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$$

Are the two required equation of the line.

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find the equation of the plane containing the it line $\frac{1}{2}(x+2) = \frac{1}{3}(y+3) = \frac{-1}{2}(z-4)$ and the point $(0,6,0)$

Required equation of the plane is

$$= \begin{vmatrix} x+2 & y+3 & z-4 \\ 2 & 3 & -2 \\ 0+2 & 6+3 & 0-4 \end{vmatrix} = 0$$

$= (\cancel{x+2)}+4)$

$$= \begin{vmatrix} x+2 & y+3 & z-4 \\ 2 & 3 & -2 \\ 2 & 9 & -4 \end{vmatrix} = 0$$

$= (x+2)(6) - (y+3)(-4) + (z-4)(12) = 0$

$= 6x + 4y + 12z - 24 = 0$

$3x + 2y + 6z - 12 = 0$

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5. Find the equation of the tangent plane at any point (α, β, γ) of the sphere
$$x^2+y^2+z^2+2ux+2vy+2wz+d=0$$

=) The point (α, β, γ) lies on the sphere

$$\alpha^2+\beta^2+\gamma^2+2u\alpha+2v\beta+2w\gamma+d=0 \quad \text{--- i}$$

The point of intersection of any line

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n} = r \quad \text{---- ii}$$

through (α, β, γ) with the given sphere are.

$$(lr+\alpha,\ mr+\beta,\ nr+\gamma)$$

Where the value of r are the roots of the quadratic equation.

$$r^2(l^2+m^2+n^2)+2r[l(\alpha+u)+m(\beta+v)+n(\gamma+w)]$$
$$+(\alpha^2+\beta^2+\gamma^2+2u\alpha+2v\beta+2w\gamma+d)=0$$

By virtue of the condition (i) one root of this quadratic equation is zero so that one of the points of intersection coincides with (α, β, γ)

In order that the second point of intersection may also coincide with (α, β, γ) the second value r must also vanish and this required

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7 Find the condition that two given straight lines:

$$\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}, \quad \frac{x-x_1}{l_2} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$$

Are coplanar. [octo-Nov-2016]

s: Equation of any plane containing the line (1) is

$$A(x-x_1) + B(y-y_1) + C(z-z_1) = 0 \quad \ldots i$$

A, B, C being number not all zero satisfying the condition.

$$Al_1 + Bm_1 + Cn_1 = 0 \quad \ldots (ii)$$

The plane (i) will contain the line (2) if

1 The point (x_2, y_2, z_2) lies on it

$$A(x_2-x_1) + B(y_2-y_1) + C(z_2-z_1) = 0 \quad \ldots iii)$$

) The point lies is perpendicular to the normal to the plane.

$$Al_2 + Bm_2 + Cn_2 = 0 \quad \ldots iv)$$

The two lines will be coplanar if the three linear homogeneous equation (ii) (iii), (iv) in A, B, C are consistent so that

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

and the equation of the plane containing the given two lines is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

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In general, the eq^n $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$

represent the plane passing through the line

$\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$ and parallel to the line $\frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}$

Similarly, $\begin{vmatrix} x-x_2 & y-y_2 & z-z_2 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$ represent the equation of the plane passing through

$\frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}$ and parallel to the line

$\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$ If these two lines are parallel ~~to the line~~ coplanar then (x_2, y_2, z_2) lies on 1st plane and (x_1, y_1, z_1) lies on second plane.

Hence the plane containing two coplanar lines is the one which passes through one line and is parallel to the other or through one line and any point on the other.

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$$l(\alpha+u)+m(\beta+v)+n(\gamma+w)=0 \quad \text{--- iii)}$$

Thus, the line

$$\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$$

meets the sphere in two coincident points at (α,β,γ) and so is a tangent line to it thereat for any set of values l,m,n which satisfy the condition (iii) and the equations (ii) of the line is

$$(x-\alpha)(\alpha+u)+(y-\beta)(\beta+v)+(z-\gamma)(\gamma+w)=0$$

$$=\alpha x+\beta y+\gamma z+u(x+\alpha)+v(y+\beta)+w(z+\gamma)+d$$

$$=\alpha^2+\beta^2+\gamma^2+2u\alpha+2v\beta+2w\gamma+d=0$$

from (i)

Which is the plane known as the tangent plane at (α,β,γ)

It follows that

$$(\alpha+u)x+(\beta+v)y+(\gamma+w)z+(u\alpha+v\beta+w\gamma+d)=0$$

is the equation of the tangent plane to the given sphere at the given point (α,β,γ)

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Q6 Find the equation of the cone whose vertex is (α, β, γ) and base $ax^2 + by^2 = 1$, $z = 0$

=) Any line through (α, β, γ) is

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n} \quad \text{---(i)}$$

This cuts $z=0$, where

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{-\gamma}{n}$$

$$= \left[\alpha - \frac{l\gamma}{n}, \beta - \frac{m\gamma}{n}, 0\right]$$

which will lie on the given conic if

$$a\left[\alpha - \frac{l\gamma}{n}\right]^2 + b\left[\beta - \frac{m\gamma}{n}\right]^2 = 1$$

Eliminating l, m, n with the help of (i), we get

$$a\left(\alpha - \frac{x-\alpha}{z-\gamma}\gamma\right)^2 + b\left(\beta - \frac{y-\beta}{z-\gamma}\right)^2 = 1$$

or

$$a(\alpha z - \gamma x)^2 + b(\beta z - z\gamma)^2 = (z-\gamma)^2$$

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8 Transforms the equation $ax+by+cz+d=0$, $a^2+b^2+c^2 \neq 0$ to the normal form:

$$lx+my+nz=0$$

Ans: As these two equation represent the same plane, we have

$$\frac{-d}{p} = \frac{a}{l} = \frac{b}{m} = \frac{c}{n} = \pm \frac{\sqrt{a^2+b^2+c^2}}{\sqrt{l^2+m^2+n^2}} = \pm\sqrt{a^2+b^2+c^2}$$

Thus, $d/p = \pm\sqrt{a^2+b^2+c^2}$. As p, according to our convention, is always positive, we shall take positive or negative sign with the radical according as, d, is negative or positive.

Thus, if d be negative, we have

$$l = -\frac{a}{\sqrt{\Sigma a^2}};\ m = -\frac{b}{\sqrt{\Sigma a^2}};\ n = \frac{-c}{\sqrt{\Sigma a^2}};\ p = +\frac{d}{\sqrt{\Sigma a^2}};$$

And if d be negative, we have

$$l = \frac{a}{\sqrt{\Sigma a^2}};\ m = \frac{b}{\sqrt{\Sigma a^2}};\ n = \frac{c}{\sqrt{\Sigma a^2}};\ p = -\frac{d}{\sqrt{\Sigma d^2}}$$

Thus, the normal form of the equation $ax+bx+cz+d=0$ is

$$\frac{-a}{\sqrt{\Sigma a^2}}x - \frac{b}{\sqrt{\Sigma a^2}}y - \frac{c}{\sqrt{\Sigma a^2}}z = \frac{d}{\sqrt{\Sigma a^2}},$$

if d be positive and

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$$+\frac{a}{\sqrt{\Sigma a^2}}x+\frac{b}{\sqrt{\Sigma a^2}}y+\frac{c}{\sqrt{\Sigma a^2}}z=\frac{-d}{\sqrt{\Sigma a^2}}$$

if d be negative.

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1) The equation of XOY plane is ......

a) $y = 0$ b) $z = 0$

c) $x = 0$ d) $z = c,\ c \neq 0$

2) If l, m, n are the direction cosines of a line then .....

a) $l^2 + m^2 + n^2 = 1$ b) $l^2 + m^2 + n^2 = 0$

c) $l + m + n = 1$ d) $l = m = n = 1$

3) The angle between the ~~line~~ planes:

$ax + by + cz + d = 0$ and $a'x + b'y + c'z + d' = 0$ is

a) $\cos^{-1}(aa' + bb' + cc')$ b) $\sin^{-1}(aa' + bb' + cc')$

c) $\cos^{-1}\left(\frac{aa' + bb' + cc'}{\sqrt{(\Sigma a^2)(\Sigma a'^2)}}\right)$ d) $\sin^{-1}\left(\frac{aa' + bb' + cc'}{\sqrt{(\Sigma a^2)(\Sigma a'^2)}}\right)$

4) The equation of the plane through the origin and parallel to the plane $3x - 4y + 5z - 6 = 0$ is

a) $3x - 4y + 5z + 6 = 0$ b) $3x + 4y - 5z + 6 = 0$

c) $3x - 4y - 5z - 6 = 0$ d) $3x - 4y + 5z = 0$

5 The value of λ for which the lines:

$\frac{x-1}{1} = \frac{y-2}{\lambda} = \frac{z+1}{-1}$ and $\frac{x+1}{-\lambda} = \frac{y+1}{2} = \frac{z-2}{1}$

are perpendicular to each other is ....

(a) 1 (b) -1 (c) 0 (d) 2

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6) The lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x}{2}=\frac{y+2}{2}=z$

Are ....

a) Skew b) Parallel

c) At right angle. d) intersecting

7 The straight line passing through (a,b,c) and parallel to the x-axis is:

a) $\frac{x-a}{0}=\frac{y-b}{0}=\frac{z-c}{1}$ b) $\frac{x-a}{1}=\frac{y-b}{0}=\frac{z-c}{0}$

c) $\frac{x-a}{1}=\frac{y-b}{1}=\frac{z-c}{0}$ d) $\frac{x-a}{0}=\frac{y-b}{1}=\frac{z-c}{1}$

8) The two equations $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ and $lx+my+nz=p$ taken together represent a

a) Circle b) Sphere

c) Plane. d) pair of planes

9) The equation of the sphere passing through $(0,0,0)$, $(a,0,0)$, $(0,b,0)$, $(0,0,c)$ is ......

a) $x^2+y^2+z^2-ax-by-cz=0$

b) $x^2+y^2+z^2+2ax+2by+2cz=0$

c) $x^2+y^2+z^2-2ax-2by-2cz=0$

d) $x^2+y^2+z^2+ax+by+cz=0$

Guiding curve of a right Circular Cylinder is

a) elipse b) any closed curve.

c) Circle d) pair of straight line.

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Proove that every equation of the first degree in x, y, z represent a plane

Proof : The most general first degree eq^n in x, y, z is

$$ax + by + cz + d = 0 \quad \text{------} \ (1)$$

Where $a, b, c, d \in R$ and a, b, c are not all zero

i.e $a^2 + b^2 + c^2 \neq 0$ .... (2)

let the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ lie on eq^n (1)

$$ax_1 + by_1 + cz_1 + d = 0 \text{ and } ax_2 + bz_2 + cz_2 + d = 0$$

Multiplying the second equation by $k \neq -1$ and then adding to the 1^{st} one we get

$$a(x_1 + kx_2) + b(y_1 + ky_2) + c(z_1 + cz_2) + d(1+k) = 0$$

$$a\frac{x_1 + kx_2}{1+k} + b\frac{y_1 + ky_2}{1+k} + c\frac{z_1 + kz_2}{1+k} + d = 0 \quad \because k \neq -1$$

This show that the point $p\Big(\frac{x_1 + kx_2}{}$

$p\left(\frac{x_1 + kx_2}{1+k} ; \frac{y_1 + ky_2}{1+k} , \frac{z_1 + kz_2}{1+k}\right)$ lies on eq^n (1)

But p is point which divides the join of P_1 and P_2 in the ratio $k:1$ Thus p lies on the line P_1P_2 for all $k \neq -1$. Hence every point on the line joining two points on eq^n (1) also lies on it.

$\therefore$ $ax + by + cz + d = 0$ represent a plane.

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find the equation of the plane in terms intercepts a, b, c which it makes on the ax

Ans: The intercept of a plane on any co-ordinate is the distance of the ~~plane~~ point where the plane meet the axis from the origin taken with the ~~appo~~ appropriate sign.

We of course, suppose here that the plane does not pass through the origin so that none of a, b, c is zero.

let the equation of plane be

$$Ax + By + Cz + D = 0 \quad \text{----} ①$$

The plane not passing through the origin we have $D \neq 0$

The point $(a,0,0), (0,b,0), (0,0,c)$ lying on the plane, eqⁿ ①, we have

$$aA + D = 0 \Rightarrow -\frac{A}{D} = \frac{1}{a}$$

$$bB + D = 0 \Rightarrow -\frac{B}{D} = \frac{1}{b}$$

$$cC + D = 0 \Rightarrow -\frac{C}{D} = \frac{1}{c}$$

The eqⁿ ① can be rewritten as.

$$-\frac{A}{D}x - \frac{B}{D}y - \frac{C}{D}z = 1$$

So that after substitution, we obtain

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1.$$

as the required eqⁿ of the plane.

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The projection of a line on the axes are 2, 3, 6. what is the length of the line?

let PQ be the length of the line and [l, m, n] be the direction cosines. Then,

Projection on x-axes, $PQ \cdot l = 2$
Projection on y-axes, $PQ \cdot m = 3$
Projection on z-axes, $PQ \cdot n = 6$

Squaring and adding, we get.

$PQ^2 (l^2 + m^2 + n^2)$

$PQ^2 = (2^2 + 3^2 + 6^2)$

$PQ^2 = 4 + 9 + 36$

$PQ^2 = 49$

$\boxed{PQ = 7}$

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Find the equation of the cone whose vertex is the point (α, β, γ) are whose generals intersect the Conic

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0, \quad z = 0$$

We have to find the eq^n of the cone is to find the locus of points on lines through A which pass through point (α, β, γ) and intersect the given curve.

The equation to any line through (α, β, γ) are

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n} \quad \text{---} \quad (1)$$

This line meet the curve $ax^2 + 2hxy + by^2 + 2gx + 2fy + c$ in the plane $z=0$ in the point given by

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n} \quad \text{i.e}$$

$$x = \alpha - \frac{l}{n}\gamma, \quad y = \beta - \frac{m}{n}\gamma, \quad z = 0$$

Which will lie on the given Conic, if

$$a\left(\alpha - \frac{l\gamma}{n}\right)^2 + 2h\left(\alpha - \frac{l\gamma}{n}\right)\left(\beta - \frac{m\gamma}{n}\right) + b\left(\beta - \frac{m\gamma}{n}\right)^2$$

$$+ 2g\left(\alpha - \frac{l\gamma}{n}\right) + 2f\left(\beta - \frac{m\gamma}{n}\right) + c = 0$$

$$\text{----} \quad (2)$$

This is the Condition for the line (1) to intersect the given conic. Eliminting l, m, n between (1) and (2) we get.

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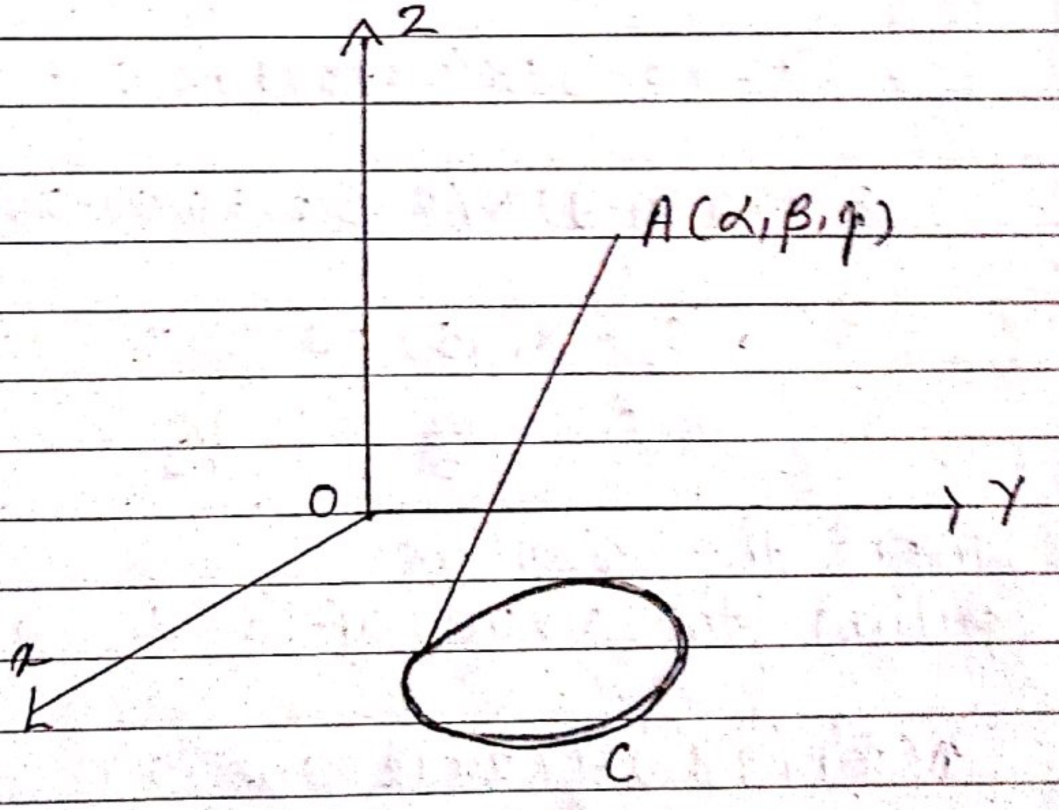
$$a\left(\alpha-\frac{x-\alpha}{z-\gamma}\right)^2+2h\left(\alpha-\frac{x-\alpha}{z-\gamma}\right)\left(\beta-\frac{y-\beta}{z-\gamma}\right)+b\left[\beta-\frac{y-\beta}{z-\gamma}\gamma\right]^2$$

$$+2g\left[\alpha-\frac{x-a}{z-\gamma}\gamma\right]+2f\left[\beta-\frac{y-\beta}{z-\gamma}\gamma\right]+c=0$$

$$= a(\alpha z-x\gamma)^2+2h(\alpha z-x\gamma)(\beta z-y\gamma)+(\beta z-y\gamma)^2$$

$$+2g(\alpha z-x\gamma)(z-\gamma)+2f(\beta z-z\gamma)(z-\gamma)$$
$$+c(z-\gamma)^2=0$$

Which is the required equation of the cone.



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Find the equation of the sphere throu the circle $x^2+y^2+z^2=9$, $2x+3y+4z$
and the point $(1,2,3)$

5: Denote

$$S = x^2+y^2+z^2 = 9$$
$$U = 2x+3y+4z-5.$$

The equation of the sphere through the Cir
$S=0$, $U=0$ is $S+KU=0$ i.e

$$x^2+y^2+z^2-9+K(2x+3y+4z-5)=0 \quad \ldots$$

If it pass through $(1,2,3)$

$$= 1^2+2^2+3^2-9+K(2x+3y+4z-5)=0$$

$$= 1+4+9-9+K(2+3(2)+4(3)-5)=0$$

$$= 5+K(15)=0$$

$$K = -\frac{15}{5} = -1/3$$

~~Then the required~~
puting the value of K in eq^n 1.

$$3(x^2+y^2+z^2)-\frac{1}{3}(2x+3y+4z-5)$$

$$3(x^2+y^2+z^2)-2x-3y-4z-22=0$$

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1) The direction cosines of the y-axis are

a) 1, 1, 1 (b) 1, 0, 0 (c) 0, 1, 0 (d) 0, 0, 1

2) The equation to a plane in normal form is

a) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ b) $\frac{x}{l} + \frac{y}{m} + \frac{z}{n} = 1$

c) $ax + by + cz' = p$ d) $lx + my + nz = p$

3) The intercept on the x-axis of the plane $x + y + z = 1$ is ....

a) 4 (b) 2 (c) 3 (d) 1

4) Any point on the line $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ is given by :

a) (α, β, γ) b) $(l\alpha, m\beta, n\gamma)$
c) $(\alpha + lr, \beta + mr, \gamma + nr)$ d) None of the above

5) The value of K so that the lines
$$\frac{x-1}{-3} = \frac{y-2}{2K} = \frac{z-3}{2}$$
$$\frac{x-1}{3K} = \frac{y-5}{1} = \frac{z-6}{-5}$$ are perpendicular to each other is ....

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a) $-10/7$ b) $-8/7$ c) $-6/7$ d)

7) The lines: $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x}{2} \ldots$ $\frac{z-3}{-2}$ are .....

a) Parallel
b) Skew
c) Intersecting
d) At right angles

7) The angle between the line $\frac{x+1}{3} = \frac{y-1}{2} = \frac{z-2}{4}$ and the plane $2x+y-3z-4=0$ is

a) $\cos^{-1}(-4\sqrt{406})$
b) $\sin^{-1}(-4/\sqrt{406})$
c) $30°$
d) None of the abo

8) The two spheres $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ and $x^2+y^2+z^2+2u'x+2v'y+2w'z+d=0$ are orthogonal if .....

a) $2(uu'+vv'+ww') = d+d'$

b) $uu'+vv'+ww' = d+d'$

c) $uu'+vv'+ww' = 2(d+d')$

d) None of the above.

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9) Equation $ax^2+by^2+cz^2+2ux+2vy+2wz+d=0$ represents a cone if ...

a) $au^2+bv^2+cw^2+d=0$

b) $\frac{u^2}{a}+\frac{v^2}{b}+\frac{w^2}{c}=d$

c) $\frac{u^2}{a}+\frac{v^2}{b}+\frac{w^2}{c}=0$

d) None of the above

10) The line which generates the surface of the cylinder is called ....

a) Axis b) Guiding line

c) Generator d) None of the above.

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2 Find the perpendicular distance of the point $p(x_1, y_1, z_1)$ from the plane

$$lx + my + nz = p.$$

ns The equation of the plane passing through the point $p(x_1, y_1, z_1)$

And given

And is parallel to the given plane is

$$lx + my + nz = p_1$$

where $lx_1 + my_1 + nz_1 = p_1$

let OKK' be the perpendicular from the origin O to the two parallel planes meeting them in K and K' so that.

$$OK = p \text{ and } OK' = p_1$$

Draw the line PL perpendicular to the given plane, we have

$$\begin{aligned} LP &= OK' - OK \\ &= p_1 - p \\ &= lx_1 + my_1 + nz_1 - p \end{aligned}$$

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Find the pole of the plane $lx+my+nz=p$ with respect to the sphere $x^2+y^2+z^2=a^2$

S:

$$lx+my+nz=p \quad \text{---} \; (1)$$

Let (α, β, γ) be the tangent pole, then we see that the equation (1) is identical with

$$\alpha x+\beta y+\gamma z=a^2 \quad \text{----} \; (2)$$

So that, on comparing (1) and (2), we obtain

$$\frac{\alpha}{l}=\frac{\beta}{m}=\frac{\gamma}{n}=\frac{a^2}{p}$$

$$\alpha=\frac{a^2 l}{p}, \quad \beta=\frac{a^2 m}{p}, \quad \gamma=\frac{a^2 n}{p}$$

Thus, the point.

$$\left[\frac{a^2 l}{p}, \frac{a^2 m}{p}, \frac{a^2 n}{p}\right]$$

is the pole of the plane $lx+my+nz=p$ w.r.t the sphere $x^2+y^2+z^2=a^2$.

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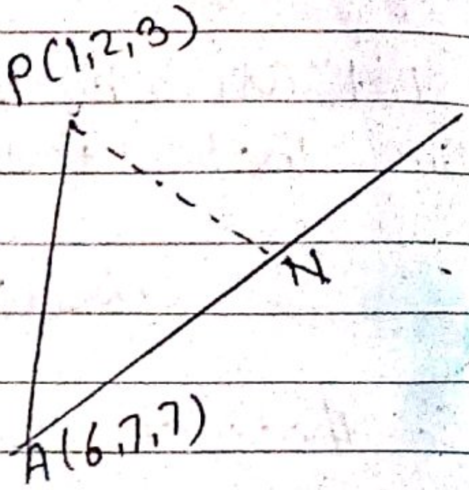
Q 4 find the perpendicular distance of $P(1,2,3)$ from the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$

Ans. $A = (6,7,7)$ will be a point on the line.
let the perpendicular from P meets the line in N.
Then



$$Ap^2 = (6-1)^2 + (7-2)^2 + (7-3)^2$$
$$= (5)^2 + (5)^2 + (4)^2$$
$$= 25 + 25 + 16$$
$$= 50 + 16$$
$$= 66$$

AN = project of ~~line~~ Ap on the given line.

$$= (6-1)\frac{3}{\sqrt{17}} + (7-2)\cdot\frac{2}{\sqrt{17}} + (7-3)\left(-\frac{2}{\sqrt{17}}\right)$$

$$= \sqrt{17}$$

$$PN^2 = Ap^2 - AN^2$$
$$PN^2 = 66 - 17$$
$$PN^2 = 49$$
$$PN = 7$$

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Find the equation of the cylinder whose generators intersect the conic
$$ax^2+2hxy+by^2+2gx+c=0$$
and the are parallel to the line
$$\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$$

ns: let (α,β,γ) be the any point on the cylinder So that the equations of the generator through the point are
$$\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n} \quad \text{---- i}$$

This lines meets the plane $z=0$ ~~is on~~ in the point given by
$$\left(\alpha-\frac{l}{n}\gamma,\ \beta-\frac{m}{n}\gamma,\ 0\right).$$

The generator eq^n ① will intersect the
en Conic, if this point lies on given conic i.e
$$a\left(\alpha-\frac{l}{n}\gamma\right)^2+b\left(\beta-\frac{m}{n}\gamma\right)^2+2h\left(\alpha-\frac{l}{n}\gamma\right)\left(\beta-\frac{m}{n}\gamma\right)$$
$$+2g\left(\alpha-\frac{l}{n}\gamma\right)+2f\left(\beta-\frac{m}{n}\gamma\right)+c=0$$

$$a(n\alpha-l\gamma)^2+b(n\beta-m\gamma)^2+2h(n\alpha-l\gamma)(n\beta-m\gamma)$$
$$+2g(n\alpha-l\gamma)n+2f(n\beta-m\gamma)n+cn^2=0$$
Generalising
α,β,γ the locus of point (α,β,γ) is
$$a(nx-lz)^2+b(ny-mz)^2+2h(nx-lz)(ny-mz)$$
$$+2n\{g(nx-lz)+f(ny-mz)\}+cn^2=0$$
This is the eq^n of cylinder

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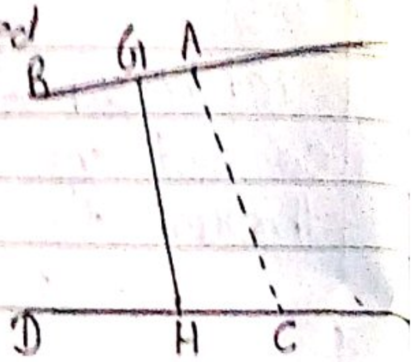
To show that the shortest distance betw. two lines lies along the line meeting them both at right angles.

Ans: Let AB, CD be two given lines.

A line is completely determined if it intersects two lines at right angle



Thus there is one and only one line which intersects the two given lines at right angles, say, at G and H.

GH is, then the shortest distance between the two lines for, if A, C be any two points, one on each of the two given lines,

- GH is the projection of AC on itself.
- $GH = AC \cos\theta$
- $GH < AC$

θ, being the angle between GH and AC.
Thus GH is the shortest distance between the two lines AB and CD.

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obtain the equation of the plane through the intersection of the planes

$x+2y+3z+4=0$ and

$4x+3y+2z+1=0$ and the origin.

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Find the perpendicular distance of the point $p(x_1, y_1, z_1)$ from the plane
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The equation of the plane passing through the point $p(x_1, y_1, z_1)$
And given
And is parallel to the given plane is

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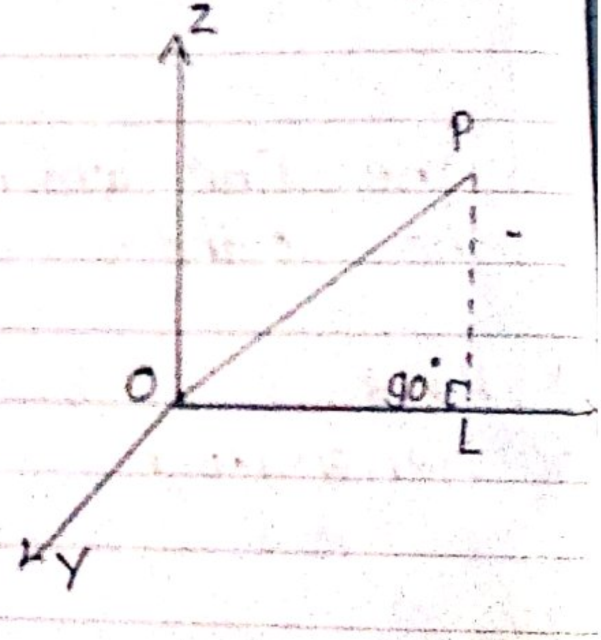
Q 1 Prove that

$$l^2+m^2+n^2=1$$

where l, m, n are direction cosines of a line.

Ans: Let op be drawn through the origin parallel to the given lines so that, l, m, n are the cosines of a angles which the line op makes with the co-ordinate axes ox, oy, oz respectively

Let (x, y, z) be the co-ordinates of any point p on this line.



Let $op = r$

$x = lr, \quad y = mr$

$z = nr.$

Squaring and adding, we obtain

$$x^2+y^2+z^2 = (l^2+m^2+n^2)r^2$$

$$r^2 = op^2 = x^2+y^2+z^2 = (l^2+m^2+n^2)r^2$$

$$= l^2+m^2+n^2 = 1$$

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